

Radio Science Investigation on a Mercury Orbiter Mission

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Abstract

We review the results from *Mariner 10* regarding Mercury's gravity field and the results from radar ranging regarding topography. We discuss the implications of improving these results, including a determination of the polar component, as well as the opportunity to perform relativistic gravity tests with a future *Mercury Orbiter*. With a spacecraft placed in orbit with periherm at 400 km altitude, apherm at 16,800 km, period 13.45 hr and latitude of periherm at +30 deg, one can expect a significant improvement in our knowledge of Mercury's gravity field and geophysical properties. The 2000 Plus mission that evolved during the European Space Agency (ESA) *Mercury Orbiter* assessment study (Hechler, 1994) can provide a global gravity field complete through the 25th degree and order in spherical harmonics. If after completion of the main mission, the periherm could be lowered to 200 km altitude, the gravity field could be extended to 50th degree and order. We discuss the possibility that a search for a Hermean ionosphere could be performed during the mission phases featuring Earth occultations.

Because of its relatively large eccentricity and close proximity to the Sun, Mercury's orbital motion provides one of the best solar-system tests of general relativity. Consequently, we emphasize the number of feasible relativistic gravity tests that can be performed within the context of the parameterized post-Newtonian formalism - a useful framework for testing modern gravitational theories. We pointed out that current results on relativistic precession of Mercury's perihelion are uncertain by 0.5 %, and we discuss the expected improvement using *Mercury Orbiter*. We discuss the importance of *Mercury Orbiter* for setting limits on a possible time variation in the gravitational constant G as measured in atomic units. Moreover, we mention that by including a space-borne ultrastable crystal oscillator (USO) or an atomic clock in the *Mercury Orbiter* payload, a new test of the solar gravitational redshift would be possible to an accuracy of one part in 10^4 with a USO, and to an accuracy of one part in 10^7 with an atomic standard. With an atomic clock and additional hardware for a multi-link Doppler system, including Doppler extraction on the spacecraft, the effect of Mercury's gravity field on USO's frequency could be measured with an accuracy of one part in 10^6 . We discuss other relativistic effects including the geodetic precession of the orbiter's orbital plane about Mercury, a planetary test of the Equivalence Principle (Nordtvedt effect), and a solar conjunction experiment to measure the relativistic time delay (Shapiro effect).

Key Words: Mercury, Mercury Orbiter, Radio Science, Mercury gravity, Mercury ionosphere, Relativistic Gravitation

1 Introduction

Mercury is the least explored of the terrestrial planets. Its known global geophysical properties include its mass, radius, and gravitational coefficients J_2 and C_{22} (Anderson *et al.*, 1987). Recently, ground-based radar ranging data and two range determinations from Mariner 10 have been used to determine both the magnitude and direction of the equatorial offset of the center of Mercury's figure from its center of mass, and to determine the orientation and major and minor axes of the ellipse which best fits Mercury's equatorial shape (Anderson *et al.*, 1995). Displacements of the center of figure from the center of mass are known for Earth (Balmino *et al.*, 1973), Moon (Kaula *et al.*, 1972; Bills and Ferrari, 1977; Smith *et al.*, 1995), Venus (Bindschadler *et al.*, 1994), and Mars (Standish, 1973; Bills and Ferrari, 1978). The global equatorial topography of Mercury from ground-based radar ranging is available (Harmon *et al.*, 1986; Harmon and Campbell, 1988; Pitjeva, 1993). The known shape and orientation of Mercury's equatorial figure and the displacement of this figure from the planet's center of mass place important constraints on the structure of Mercury's interior. The magnitude of the center of figure offset implies an excess crustal thickness of 12 km or less, comparable to the Moon's excess. By comparing the equatorial ellipticity with the Mariner 10 gravity coefficient C_{22} , and assuming Airy isostatic compensation, Anderson *et al.* (1995) conclude that Mercury's crustal thickness is in the range 100 to 250 km. A similar calculation for the Moon, using $C_{22} = 2.2 \times 10^{-5}$ and $(a - b)/a = 1.4 \times 10^{-3}$, gives a mean crustal thickness of about ~ 72 km (Schubert *et al.*, 1995). Because the radar ranging data are restricted between 12° and -10° in latitude, *Mercury Orbiter* can provide additional information on geophysical parameters aligned with the polar axis. It can also provide a global gravity field complete through the 25th degree and order in spherical harmonics (A polar orbit with periherm at 400 km altitude, apherm at 16,800 km, period = 13.45 hr, latitude of periherm at $+30^\circ$). If after completion of the main mission, the periherm could be lowered to 200 km altitude, the gravity field could be extended to 50th degree and order.

Radio propagation measurements using spacecraft signals have also provided scientific information on the atmospheres, ionospheres, rings, tori, and surfaces of essentially every planetary system encountered in the brief history of solar system exploration (e.g. Tyler, 1987). A search for a Hermean ionosphere could be performed with a *Mercury Orbiter* during the mission phases featuring Earth occultations.

Because of its relatively large eccentricity and close proximity to the Sun, Mercury's orbital motion provides one of the best solar-system tests of general relativity. Its excess perihelion precession, amounting to 42.98 arcseconds per century, is one of the three classical tests of the theory. Coherent Doppler and ranging measurements using a *Mercury Orbiter* could provide improved orbital information and hence, more sensitive tests. A number of predictions of alternative gravitational theories might be detected as well, or failing a detection, tighter constraints could be placed on permissible deviations from general relativity.

By including a space-borne ultrastable crystal oscillator (USO) or an atomic clock in the *Mercury Orbiter* payload, a new test of the gravitational redshift would be possible. The effect of solar gravity on the oscillator's frequency could be measured to an accuracy of one part in 10^4 with a USO, and to an accuracy of one part in 10^7 with an atomic standard. With an atomic clock and additional hardware for a multi-link Doppler system, including Doppler extraction on the spacecraft, the effect of Mercury's gravity field could be measured with an accuracy of one part in 10^6 .

This paper summarizes and discusses the different radio science experiments proposed for a future *Mercury Orbiter* mission. The outline of the paper is as follows: Section II discusses a present knowledge of Mercury's gravity field and future improvements offered by *Mercury Orbiter*. In section III, we discuss the possibility of a search for a Hermean ionosphere. Mercury ranging data are reviewed in section IV. In section V, we discuss feasible relativistic gravitational tests using the satellite in orbit around Mercury. In particular, we present the parameterized post-Newtonian formalism, which has become a useful

theoretical framework for analyzing the gravitational experiments within the solar system. This section contains a catalog of relativistic gravitational experiments possible with a *Mercury Orbiter*. We present both the quantitative and qualitative analyses of the measurable effects. In section VI, we discuss briefly the radio science instrumentation required for the *Mercury Orbiter* mission under consideration by ESA.

2 Mercury Gravity Field

The determination of Mercury’s gravity field could be obtained by the reduction of Doppler radio tracking data. This is an old and proven technique used for the moon, Mars and Venus (Konopliv and Sjogren, 1994). The field would be modeled with spherical harmonic coefficients providing a smooth global model. Given the power of modern computers, it is feasible to estimate literally thousands of harmonic coefficients. The resulting coefficients would be interpreted in terms of a gravity surface function, which would be plotted as a surface gravity map. For other planets, in particular Venus, Earth, the moon and Mars, the correlation of gravity maps with surface topography and imaging has revealed the isostatic state of surface features. For example, similar features at different locations on a planet can have significantly different internal density variations or structure. Geophysical interpretation can yield information on the distribution of crustal thickness and dynamic hot spots. The degree of isostatic compensation may indicate an old crust, nearly completely compensated, or a crust with sufficient rigidity to imply a cold interior.

For Mercury, the second order gravity coefficients C_{20} , C_{21} , S_{21} , C_{22} and S_{22} would provide the orientation of the axes of the moments of inertia, which would be compared with the spin axis orientation. A more accurate C_{20} coefficient would provide a better-determined gravitational oblateness, and C_{22} coefficient would reveal a better-determined gravitational ellipticity of the equator. At a more local level, all the other coefficients, to as high a degree and order as possible, would provide a gravity map for comparison with surface topography. The most obvious and interesting correlations would be negative (i.e., gravity highs in topographic lows or gravity lows in topographic highs). On the moon and Mars there are negative correlations for some large basins (but not all). This has placed strong constraints on interior thermal models. We anticipate that a gravity map of Mercury’s Coloris Basin would place significant constraints on its underlying internal structure.

As a rule of thumb, the spatial resolution of gravity features derived from coherent X-band Doppler data would be less than the spacecraft altitude. For a 400 km altitude perihelion, the best resolution would be approximately 300 km or 7.1° on the surface of Mercury and would correspond to a 25th degree and order spherical harmonic model. At a lower 200 km altitude a 50th degree and order model could be obtained for a total of 2600 harmonic coefficients. The ideal data acquisition period would be 59 days of Doppler data covering at least ± 1 hour of periapsis. Data within 5° to 10° of solar conjunction would be too noisy for gravity field determination. During gravity passes other non-gravitational forces would be minimized. These data would provide complete global coverage. Data over 59 days at a different viewing geometry would significantly strengthen the solution.

In addition to the gravity coefficients, other parameters would be estimated simultaneously from the coherent Doppler data. These include the Love number k_2 , which would provide a measure of whether there is a solid or liquid core, translational forces from solar radiation impinging on spacecraft parts, Mercury’s albedo, corrections to the Mercury ephemeris, and location of the spin pole and rotation rate.

The final data products that would be produced for geophysical interpretation are:

1. The spherical harmonic coefficients.
2. A full covariance matrix on all the coefficients (i.e., $1 - \sigma$ uncertainties and their correlations).
3. A $1^\circ \times 1^\circ$ digitized vertical surface gravity map.
4. A $1^\circ \times 1^\circ$ digitized geoid map.

5. $1 - \sigma$ uncertainty maps on gravity and geoid.
6. If topography data, $1^\circ \times 1^\circ$ Bouguer gravity maps.
7. $1^\circ \times 1^\circ$ Isostatic Anomaly map.

3 Mercury Ionosphere

Mercury was encountered three times by the spacecraft *Mariner 10*, but was occulted by the planet's disk only at the first flyby on 29 March 1974. The S/X-band dual-frequency radio links from spacecraft to Earth were monitored during the flyby (Howard *et al.*, 1974). The attempt to detect the ionosphere from the dispersive phase shift between the two downlinks, however, yielded only an upper limit of 1000 cm^{-3} for the electron density in the altitude range from the surface up to 1000 km (Fjeldbo *et al.*, 1976). These observations were used to infer an upper limit to the surface density of the Hermean atmosphere of $10^6 \text{ molecules cm}^{-3}$. Taking a surface temperature of 500 K, this corresponds to a pressure of ca. 10^{-13} bar (a very decent vacuum!). The mission profile that evolved during the ESA *Mercury Orbiter* assessment study (Hechler, 1994) offers definite prospects for radiometric investigations. The geometry has many parallels to that designed for *Mars Orbiter*, i.e. regularly recurring occultations at polar latitudes during certain phases of the mission (Tyler *et al.*, 1992). Whereas the atmosphere on Mars is dense enough to be synoptically measured on a global basis, Mercury's high temperature and low surface gravity combine to produce only a residual "exosphere" with the above mentioned diminutive surface densities. Detecting an ionosphere on Mercury using radio occultation techniques is a difficult, but not hopeless, task. Generally speaking, the ionosphere should be easier to detect than the neutral atmosphere. One classical example of an ionospheric detection was the *Pioneer 10* measurement during occultation of the Jovian moon Io (Kliore *et al.*, 1974), for which peak densities of $6 \times 10^4 \text{ electrons cm}^{-3}$ were derived. The geometry of this particular occultation enabled the detection to be made with only an S-band downlink. The ionospheric effect on the S-band downlink during the *Voyager 2* occultation of Neptune's moon Triton was about five times greater than that of the neutral atmosphere (Tyler *et al.*, 1989). There are, of course, counterexamples such as Titan (Lindal *et al.*, 1983), which produced dramatic refraction effects from its dense, extended atmosphere, but no discernable ionospheric signature. Radio propagation experiments on previous ESA interplanetary missions (e.g. *Giotto*, *Ulysses*) have utilized the extensive facilities of the NASA Deep Space Network. It is expected that similar ground equipment will be available for the *Mercury Orbiter*. The assessment study for the *Mercury Orbiter* recommended an S/X-Band radio communications system for both up- and downlinks (ESA Study Team, 1994). For the dual-frequency (S/X-band) downlink configuration on *Ulysses*, the smallest detectable change in differential phase (i.e. S-band minus 3/11 X-band) was estimated to be ca. 5° at S-band, corresponding to an electron content of about $10^{14} \text{ electrons m}^{-2}$ (Bird *et al.*, 1992). The occultation geometry afforded by an orbiting spacecraft, however, for which the lateral motion of the signal ray path through the ionized medium is quite rapid, may still enable detection of intervening plasma without the dual-frequency capability. For a geometry of this type, the smallest phase change measureable is estimated as $\sim 0.5^\circ$. This is also the level of detectability attributed to with the *uplink* radio occultation instrumentation presently being developed for the *Pluto Express* Mission [G.L. Tyler, 1995, private communication]. This tiny change in phase, which provides a fair estimate of the present-day detection threshold, corresponds to a neutral column density of $5 \times 10^{24} \text{ molecules m}^{-2}$ or an electron columnar content of $10^{14} \text{ electrons m}^{-2}$ at X-band. For a spherically symmetric Hermean atmosphere (ionosphere), this can be shown to translate to peak number densities along the ray path at the planet's limb of $5 \times 10^{12} \text{ molecules cm}^{-3}$ ($100 \text{ electrons cm}^{-3}$). Using the upper limits quoted by Fjeldbo *et al.* (1976), column densities for the ray path tangent to the limb may be estimated for Mercury. The neutral column density is found to be $10^{18} \text{ molecules m}^{-2}$, which is, unfortunately, about six orders of magnitude below the detection threshold. Phase changes induced by

the neutral gas are thus probably negligible. The ionized component, however, if we assume the peak reaches 10^3 electrons cm^{-3} , can produce a content of 10^{15} electrons m^{-2} , which would place it above the lower limits of detectability. This estimate, it should be emphasized, is only valid for a global-scale ionosphere. However, as pointed out during the *Mercury Orbiter* assessment study (ESA Study Team, 1994), atomic sodium, which emanates from various hot spots on the planet’s surface, may well be the most prevalent component of the Hermean exosphere. Mean surface densities of up to 5×10^4 cm^{-3} are inferred (Cheng *et al.*, 1987). The Na atom is photoionised on time scales of a few hours, so this is presumed to be the most important source of ionospheric plasma. The proximity of the magnetopause and the tailward-streaming solar wind, however, may cause many ions to be swept away at rates even quicker than those of the sources. If the electron density is concentrated in the atmosphere above a local source, densities considerably higher than 1000 cm^{-3} would be necessary to produce a measureable effect. Regularly recurring Earth occultations, during which radio tracking at ground-based stations is interrupted, are included in virtually every *Mercury Orbiter* mission scenario because they are basically “unavoidable”. Two such occultation intervals, one near inferior conjunction and one near superior conjunction, were found to occur for the prototype *Mercury Orbiter* mission (Hechler, 1994). The total number of such occultations, each with ingress and egress opportunities, is well over 100.

Another feature, as mentioned above, is the rapid swath taken by the radio ray path through the ionosphere. This magnifies the plasma Doppler shift on the occulted radio signal, thereby increasing the detection probability. Synoptic Doppler observations of ionospheric electron densities on a *Mercury Orbiter* would thus appear to be very difficult, but still worth a good look. Even in the absence of a permanent global ionosphere it is still possible, however, that a systematic examination of the radio occultation data would result in the serendipitous detection of near-surface plasma clouds.

4 Mercury Ranging Data

A summary and discussion of data used in the fundamental ephemerides DE200/LE200 is available (Standish, 1990; Standish *et al.*, 1992). Those data include Mercury radar ranging data spanning the years 1966-1974. Analysis of the older data plus additional data spanning the years 1974 to 1990 is underway (Anderson *et al.*, 1995). Included are two range fixes from the *Mariner 10* Mercury flybys on March 29, 1974 and March 16, 1975 (Anderson *et al.*, 1987). The current JPL set of reduced Mercury radar ranging data is summarized in Table 1. The Goldstone data are from 34 m and 70 m stations located in California’s Mojave desert, the Arecibo data are from Puerto Rico, while the Haystack data are from Tyngsboro, Massachusetts.

The data used in current relativity tests are reduced ranging measurements to Mercury’s surface. Both radar time delay and Doppler data have been used in the reduction (for an explanation of the Doppler-delay technique see e.g. Ingalls and Rainville 1972; Shapiro *et al.*, 1972; Harmon *et al.*, 1986). The 1978-1982 reduced data are from archives at the Harvard Smithsonian Center for Astrophysics (J. Chandler, private communication). Systematic errors in radar ranging data limit their effectiveness for tests of general relativity, most notably the excess precession of Mercury’s perihelion, and topography models have been introduced for purposes of minimizing the error (Anderson *et al.*, 1991; Pitjeva, 1993; Anderson *et al.*, 1995). New data, much less affected by systematic error, will become available with *Mercury Orbiter*. Specifically, transponded ranging, similar to that from the *Mariner 9 Mars Orbiter*, will be available. Eventually, with the introduction of a *Mercury Lander*, even better surface transponded ranging will become available, similar to ranging data from the two *Viking Landers*. Unfortunately, no *Venus Orbiter* has carried a ranging transponder to date. However, we have removed Venus topography from Venus radar ranging data by using the *Pioneer 12* radar altimetry measurements, and one range fix is available from the *Galileo* spacecraft flyby in 1990 (Anderson *et al.*, 1991). In addition, the MGN X-band Doppler data provide a measure of the line-of-sight velocity to an accuracy of 0.02 mm/s (or

$\approx 10\text{m}$ range accuracy). Of all the inner planets, only Mercury currently requires a parameterized topography model for ranging data analysis. The *Mercury Orbiter* will remove this restriction, and the orbital motion of Mercury will be placed on an equal footing with Venus and Mars.

5 Relativistic Gravitational Tests

Mercury is the closest to the Sun of all the planets of the terrestrial group and, due to its unique location and orbital parameters, it is especially suited to relativistic gravitational experiments. The short period of its solar orbit will allow experiments over several orbital revolutions. In this section we will define the list of gravitational experiments for the *Mercury Orbiter* mission. Depending on the nature of the measurable effect, there are two main groups of possible experiments, namely: (i) the celestial mechanical studies of the orbital motion and rotation of Mercury in the gravitational field of the Sun, as well as observations of the motion of the *Mercury Orbiter* in the solar gravitational field at the vicinity of the planet Mercury, and (ii) the studies of the gravitational influence on the propagation of the radio waves along the Earth-Mercury path. We will discuss separately the possible experiments from both these groups and will present the estimates of the magnitude and physical character of the measured effects. Analysis, performed in this section, is directed towards the future mission, so we will show which relativistic effects can be measured, and how accurately these measurements can be done.

5.1 Parameterized Post-Newtonian Gravity.

Metric theories of gravity have a special position among all the other possible theoretical models. The reason is that, independently of the many different principles at their foundations, the gravitational field in these theories affects the matter directly through the metric tensor of the Riemannian space-time g_{mn} , which is determined from the field equations of a particular theory of gravity. As a result, in contrast to Newtonian gravity, this tensor contains the properties of a particular gravitational theory as well as carries the information about the gravitational field of the bodies. This property of the metric tensor enables one to analyze the motion of matter in one or another metric theory of gravity based only on the basic principles of modern theoretical physics.

In order to accumulate the features of many modern metric theories of gravity in one theoretical scheme as well as to create the ‘versatile instrument’ to plan the gravitational experiments and to analyze the data obtained, Nordtvedt and Will have proposed the parameterized post-Newtonian (PPN) formalism (Nordtvedt 1968a; Will 1971; Will & Nordtvedt 1972). This formalism allows one to describe within the common framework the motion of the celestial bodies for a wide class of metric theories of gravity. Within the accuracy of modern experimental techniques, the PPN formalism provides a useful foundation for testing the predictions of different metric theories of gravity within the weak gravitational field and slow motion approximation appropriate for the solar system.

The metric tensor of the general Riemannian space-time in the PPN formalism is generated by some given distribution of matter in the form of an ideal fluid. It is represented by a sum of gravitational potentials with arbitrary coefficients, the PPN parameters. The most general form of this metric tensor in four dimensions may be written as follows¹:

$$g_{00} = 1 - 2U + 2(\beta + \tau)U^2 - (2\gamma + 2 + \alpha_3 + \zeta_1)\Phi_1 + \zeta_1 A + 2(\zeta + \tau)\Phi_w - \\ - 2\left[(3\gamma + 1 - 2\beta + \tau + \zeta_2)\Phi_2 + (1 + \zeta_3)\Phi_3 + 3(\gamma + \zeta_4)\Phi_4 + \nu\chi_{,00}\right] -$$

¹The geometrical units $\hbar = c = G = 1$ are used throughout as is the metric convention (+ ---).

$$-(\alpha_1 - \alpha_2 - \alpha_3)w_\mu w^\mu U + \alpha_2 w^\mu w^\nu U_{\mu\nu} + (\alpha_1 - 2\alpha_3)w^\mu V_\mu + \mathcal{O}(c^{-6}), \quad (1a)$$

$$g_{0\alpha} = \frac{1}{2}(4\gamma + 3 + 2\nu + 2\tau + \alpha_1 - \alpha_2 + \zeta_1)V_\alpha + \\ + \frac{1}{2}(1 - 2\nu - 2\tau + \alpha_2 - \zeta_1)W_\alpha - \frac{1}{2}(\alpha_1 - 2\alpha_2)w_\alpha U + \alpha_2 w^\mu U_{\alpha\mu} + \mathcal{O}(c^{-5}), \quad (1b)$$

$$g_{\alpha\beta} = \gamma_{\alpha\beta}(1 + 2(\gamma + \tau)U) + 2\tau U_{\alpha\beta} + \mathcal{O}(c^{-4}). \quad (1c)$$

with the generalized gravitational potentials given as follows:

$$U(z^p) = \int \frac{\rho_0(z'^p)}{|z^\nu - z'^\nu|} d^3 z'^\nu, \quad V^\alpha(z^p) = - \int \frac{\rho_0(z'^p) v^\alpha(z'^p)}{|z^\nu - z'^\nu|} d^3 z'^\nu, \\ W^\alpha(z^p) = \int \rho_0(z'^p) v_\mu(z'^p) \frac{(z^\alpha - z'^\alpha)(z^\mu - z'^\mu)}{|z^\nu - z'^\nu|} d^3 z'^\nu, \\ A(z^p) = \int \rho_0(z'^p) \frac{[v^\mu(z'^p)(z^\mu - z'^\mu)]^2}{|z^\nu - z'^\nu|^3} d^3 z'^\nu, \quad \chi(z^p) = \int \rho_0(z'^p) |z^\nu - z'^\nu| d^3 z'^\nu, \\ U^{\alpha\beta}(z^p) = \int \rho_0(z'^p) \frac{(z^\alpha - z'^\alpha)(z^\beta - z'^\beta)}{|z^\nu - z'^\nu|^3} d^3 z'^\nu, \\ \Phi_1(z^p) = - \int \frac{\rho_0(z'^p) v_\lambda(z'^p) v^\lambda(z'^p)}{|z^\nu - z'^\nu|} d^3 z'^\nu, \quad \Phi_2(z^p) = \int \frac{\rho_0(z'^p) U(z'^p)}{|z^\nu - z'^\nu|} d^3 z'^\nu, \\ \Phi_3(z^p) = \int \frac{\rho_0(z'^p) \Pi(z'^p)}{|z^\nu - z'^\nu|} d^3 z'^\nu, \quad \Phi_4(z^p) = \int \frac{\rho_0(z'^p) p(\rho(z'^p))}{|z^\nu - z'^\nu|} d^3 z'^\nu, \\ \Phi_w(z^p) = \int \int \rho_0(z'^p) \rho_0(z''^p) \frac{(z^\beta - z'^\beta)}{|z^\nu - z'^\nu|^3} \left[\frac{(z_\beta - z''^\beta)}{|z'^\nu - z''^\nu|} - \frac{(z'^\beta - z''^\beta)}{|z^\nu - z''^\nu|} \right] d^3 z'^\nu d^3 z''^\nu. \quad (2)$$

Besides the PPN parameters $(\gamma, \beta, \zeta, \alpha_1 - \alpha_3; \zeta_1 - \zeta_4)$, the expression (1) contains two other parameters ν and τ (Denisov and Turyshev, 1990; Turyshev, 1990). The parameter ν reflects the specific choice of the gauge conditions and for the standard PPN gauge it is given as $\nu = 0$, but for harmonic gauge conditions one should choose $\nu = \frac{1}{2}$. The parameter τ describes the possible anisotropy of the space-time and it corresponds to a different spatial coordinates, which may be chosen for modelling the experimental situation. For example, the case $\tau = 0$ corresponds to harmonic coordinates, while $\tau = -1$ corresponds to the standard (Schwarzschild) coordinates. Then, a particular metric theory of gravity in the PPN formalism with a specific coordinate gauge (ν, τ) might be fully characterized by the means of the ten PPN parameters. This formalism uniquely prescribes the values of these parameters for each particular theory under study. In the standard PPN gauge (i.e. in the case when $\nu = \tau = 0$) these parameters have clear physical meaning. The parameter γ represents the measure of the curvature of the space-time created by the unit rest mass; the parameter β is the measure of the non-linearity of the law of superposition of the gravitational fields in a theory of gravity (or the measure of the metricity); the parameter ζ quantifies the violation of the local position invariance; the group of parameters $\alpha_1, \alpha_2, \alpha_3$ specify the violation of Lorentz invariance (or the presence of the privileged reference frame), and, finally, the parameters $\zeta_1, \zeta_2, \zeta_3, \zeta_4$ reflect the violation of the law of total momentum conservation for a closed gravitating system. Note that general relativity, when analyzed in standard PPN gauge, gives: $\gamma = \beta = 1$

and all the other eight parameters vanish. Whereas for the Brans-Dicke theory, $\beta = 1, \gamma = \frac{1+\omega}{2+\omega}$, where ω is an unknown dimensionless parameter of this theory.

The main properties of the solution (1)-(2) are well established and widely in use in modern astronomical practice (Moyer, 1971; Moyer, 1981; Brumberg, 1991; Standish *et al.*, 1992; Will, 1993). For practical purposes one chooses the inertial reference frame located in the center of mass of an isolated distribution of matter, then, by performing a power expansion of the potentials in terms of spherical harmonics, one obtains the post-Newtonian definitions of the mass of the body m_a , its center of mass \mathcal{Z}_a^α , momentum \mathcal{P}_a^α and angular momentum $S_a^{\alpha\beta}$. Thus the definitions for the mass m_a and coordinates of the center of mass of the body \mathcal{Z}_a^α in any inertial reference frame are given by the formulae (for more detailed analysis see (Damour, 1983) and (Will, 1993) and references therein):

$$m_a = \int_a d^3 z_a'^\epsilon \rho_{a0}^* \left(1 + \Pi + \frac{1}{2}U - \frac{1}{2}v_\lambda v^\lambda + \mathcal{O}(c^{-4}) \right), \quad (3a)$$

$$\mathcal{Z}_a^\alpha(t) = \frac{1}{m_a} \int_a d^3 z_a'^\epsilon \rho_{a0}^* z_a'^{\alpha} \left(1 + \Pi + \frac{1}{2}U - \frac{1}{2}v_\lambda v^\lambda + \mathcal{O}(c^{-4}) \right). \quad (3b)$$

where the conserved mass density is defined by $\rho_{a0}^* = \rho_{a0} \sqrt{-g} u^0$.

It has been shown (Fock, 1959; Ni & Zimmerman, 1978) that for an isolated distribution of matter in this approximation there exist a set of inertial reference frames and ten integrals of motion corresponding to ten conservation laws. Moreover, Ni and Zimmerman have shown (1978) that in order for a metric theory to have the total energy-momentum conservation laws (so-called fully conservative theories), only three PPN parameters may have non-zero means, namely γ, β and ζ . For the future analysis of gravitational experiments on *Mercury Orbiter*, we will limit ourselves to this class of metric theories of gravity.

In the post-Newtonian approximation the mass m_a (3a) is conserved and the centre of mass \mathcal{Z}_a^α moves in space with a constant velocity along a straight line. Thus, $\mathcal{Z}_a^\alpha(t) = \mathcal{P}^\alpha \cdot t + \mathcal{K}^\alpha$, where the constants $\mathcal{P}^\alpha = d\mathcal{Z}_B^\alpha/dt$ and \mathcal{K}^α are the body's momentum and center of inertia, respectively. One can choose from the set of inertial reference frame the barycentric inertial one. In this frame the functions \mathcal{Z}^α must equal zero for any moment of time. This condition can be satisfied by applying to the metric eqs.(1) the post-Galilean transformations (Chandrasekhar & Contopoulos, 1967) where the constant velocity u^α and displacement of origin b^α should be selected in a such a way that \mathcal{P}^α and \mathcal{K}^α equal zero (for details see Kopejkin, 1988; Will, 1993). The resulting barycentric inertial reference frame has been adopted for the fundamental planetary and lunar ephemerides (Newhall *et al.*, 1983; Standish *et al.*, 1992). Moreover, the coordinate time of the solar barycentric (harmonic) reference frame is the TDB time scale, which has been adopted in modern astronomical practice (Fukushima, 1995).

In order to analyze the motion of bodies in the solar system barycentric reference frame one may obtain the Lagrangian function L_N describing the motion of N extended self-gravitating bodied (Turyshchev, 1990; Will, 1993). Within the accuracy necessary for our future discussion of the gravitational experiments on the *Mercury Orbiter* mission, this function can be presented in the following form:

$$\begin{aligned} L_N = & \sum_a^N \frac{m_a}{2} v_{a\mu} v_a^\mu \left(1 - \frac{1}{4} v_{a\mu} v_a^\mu \right) - \\ & - \sum_a^N \sum_{b \neq a}^N \frac{m_a m_b}{r_{ab}} \left(\frac{1}{2} + (3 + \gamma - 4\beta + 3\zeta) \Omega_a - \zeta n_{ab\lambda} n_{ab\mu} \Omega_a^{\lambda\mu} - \right. \\ & \left. - (\gamma + \tau + \frac{1}{2}) v_{a\mu} v_a^\mu + (\gamma + \tau + \frac{3}{4}) v_{a\mu} v_b^\mu - \right. \end{aligned}$$

$$\begin{aligned}
& -\left(\frac{1}{4} - \tau\right)n_{ab\lambda}n_{ab\mu}v_a^\lambda v_b^\mu - \tau(n_{ab\mu}v_a^\mu)^2 + \\
& + \frac{n_{ab\lambda}}{r_{ab}}\left[(\gamma + \frac{1}{2})v_{a\mu} - (\gamma + 1)v_{b\mu}\right]S_a^{\mu\lambda} + n_{ab\lambda}n_{ab\mu}\frac{I_a^{\lambda\mu}}{r_{ab}^2} + \\
& + (\beta + \tau - \frac{1}{2})\sum_a^N m_a\left(\sum_{b \neq a}^N \frac{m_b}{r_{ab}}\right)^2 + \\
& + (\zeta + \tau)\sum_a^N \sum_{b \neq a}^N \sum_{c \neq a,b}^N m_a m_b m_c \left[\frac{n_{ab\lambda}}{2r_{ab}^2}(n_{bc}^\lambda + n_{ca}^\lambda) - \frac{1}{r_{ab}r_{ac}}\right] + \sum_a^N m_a \mathcal{O}(c^{-6}), \tag{4}
\end{aligned}$$

where m_a is the isolated rest mass of a body a , the vector r_a^α is the barycentric radius-vector of this body, the vector $r_{ab}^\alpha = r_b^\alpha - r_a^\alpha$ is the vector directed from body a to body b , and the vector $n_{ab}^\alpha = r_{ab}^\alpha/r_{ab}$ is the usual notation for the unit vector along this direction. It should be noted that the expression (4) does not depend on the parameter ν , which confirms that this parameter is the gauge parameter only. The tensor $I_a^{\mu\nu}$ is the spatial trace-free (STF) (Thorne, 1980) tensor of the quadrupole moment of body a defined by

$$I_a^{\mu\nu} = \frac{1}{2m_a} \int_a d^3 z_a'^\nu \rho_{a0}^*(z_a'^p) \left(3z_a'^\mu z_a'^\nu - \gamma^{\mu\nu} z_a'^\lambda z_a'^\lambda\right) = 3J_{a2}^{\mu\nu} - \gamma^{\mu\nu} J_{a2}, \tag{5}$$

where the quantity J_{a2} is the quadrupole coefficient. The tensor $S_a^{\mu\nu}$ is the body's intrinsic STF spin moment which is given as:

$$S_a^{\mu\nu} = \frac{1}{m_a} \int_a d^3 z_a'^\nu \rho_{a0}^*(z_a'^p) [u_a^\mu z_a'^\nu - u_a^\nu z_a'^\mu], \tag{6}$$

where u_a^μ is the rotational velocity of body a . Finally, the tensor $\Omega_a^{\mu\nu}$ is the body's gravitational binding energy tensor:

$$\Omega_a^{\alpha\beta} = -\frac{1}{2m_a} \int_a \int_a d^3 z_a'^\nu d^3 z_a''^\nu \rho_0(z_a'^p) \rho_0(z_a''^p) \frac{(z_a'^\alpha - z_a''^\alpha)(z_a'^\beta - z_a''^\beta)}{|z_a'^\nu - z_a''^\nu|^3}. \tag{7}$$

Furthermore, with the help of the Lagrangian (4), one can obtain the corresponding equations of motion of an arbitrary extended body a as follows:

$$\begin{aligned}
\ddot{r}_a^\alpha &= \sum_{b \neq a} \frac{\mathcal{M}_{b\beta}^\alpha}{r_{ab}^2} \hat{n}_{ab}^\beta + \sum_{b \neq a} \frac{m_b}{r_{ab}^2} \left[\mathcal{A}_{ab}^\alpha + \frac{\mathcal{B}_{ab}^\alpha}{r_{ab}} + \frac{\mathcal{C}_{ab}^\alpha}{r_{ab}^2} - \right. \\
& - \frac{n_{ab}^\alpha}{r_{ab}} \left((2\beta + 2\gamma + 2\tau + 1)m_a + (2\beta + 2\gamma + 2\tau)m_b \right) \left. \right] + \\
& + \sum_{b \neq a} \sum_{c \neq a,b} m_b m_c \mathcal{D}_{abc}^\alpha + \mathcal{O}(c^{-6}), \tag{8}
\end{aligned}$$

where, in order to account for the influence of the gravitational binding energy $\Omega_b^{\alpha\beta}$, we have introduced in the equation (8) the tensor of passive gravitational rest mass $\mathcal{M}_b^{\alpha\beta}$ as follows

$$\mathcal{M}_b^{\alpha\beta} = m_b \left[\delta^{\alpha\beta} \left(1 + (3 + \gamma - 4\beta + 3\zeta)\Omega_b - 3\zeta n_{ab\lambda} n_{ab\mu} \Omega_b^{\lambda\mu} \right) + 2\zeta \Omega_b^{\alpha\beta} + \mathcal{O}(c^{-4}) \right], \tag{9}$$

and the unit vector n_{ab} has also been corrected by the gravitational binding energy and the tensor of the quadrupole moment $I_a^{\alpha\beta}$ of the body a under question:

$$\begin{aligned}\hat{n}_{ab}^\alpha &= n_{ab}^\alpha \left(1 + (3 + \gamma - 4\beta + 3\zeta)\Omega_a - 3\zeta n_{ab\lambda} n_{ab\mu} \Omega_a^{\lambda\mu} + 5n_{ab\lambda} n_{ab\mu} \frac{I_a^{\lambda\mu}}{r_{ab}^2} \right) + \\ &+ 2\zeta n_{ab\beta} \Omega_a^{\alpha\beta} + 2n_{ab\beta} \frac{I_a^{\alpha\beta}}{r_{ab}^2} + \mathcal{O}(c^{-4}).\end{aligned}\quad (10)$$

The term \mathcal{A}_{ab}^α in the expression (8) is the orbital term which is given as follows:

$$\begin{aligned}\mathcal{A}_{ab}^\alpha &= v_{ab}^\alpha n_{ab\lambda} \left(v_a^\lambda - (2\gamma + 2\tau + 1)v_{ab}^\lambda \right) + \\ &+ n_{ab}^\alpha \left(v_{a\lambda} v_a^\lambda - (\gamma + 1 - \tau)v_{ab\lambda} v_{ab}^\lambda + 3\tau(n_{ab\lambda} v_{ab}^\lambda)^2 - \frac{3}{2}(n_{ab\lambda} v_b^\beta)^2 \right),\end{aligned}\quad (11)$$

The spin-orbital term \mathcal{B}_{ab}^α has the form:

$$\begin{aligned}\mathcal{B}_{ab}^\alpha &= \left(\frac{3}{2} + 2\gamma \right) v_{ab\lambda} (S_a^{\alpha\lambda} + S_b^{\alpha\lambda}) + \frac{1}{2} v_{a\lambda} (S_a^{\alpha\lambda} - S_b^{\alpha\lambda}) + \\ &+ \frac{3}{2} (1 + 2\gamma) n_{ab\lambda} v_{ab\beta} \left[n_{ab}^\beta (S_a^{\alpha\lambda} + S_b^{\alpha\lambda}) - n_{ab}^\alpha (S_a^{\beta\lambda} + S_b^{\beta\lambda}) \right] + \\ &+ \frac{3}{2} n_{ab\lambda} \left[n_{ab}^\alpha (v_{a\beta} S_b^{\beta\lambda} - v_{b\beta} S_a^{\beta\lambda}) + n_{ab\beta} v_{ab}^\beta S_b^{\alpha\lambda} \right].\end{aligned}\quad (12)$$

The term \mathcal{C}_{ab}^α is caused by the oblateness of the bodies in the system:

$$\mathcal{C}_{ab}^\alpha = 2n_{ab\beta} I_b^{\alpha\beta} + 5n_{ab}^\alpha n_{ab\lambda} n_{ab\mu} I_b^{\lambda\mu}. \quad (13)$$

And, finally, the contribution \mathcal{D}_{abc}^α to the equations of motion (8) of body a , caused by the interaction of the other planets ($b \neq a, c \neq a, b$) with each other is:

$$\begin{aligned}\mathcal{D}_{abc}^\alpha &= \frac{n_{ab}^\alpha}{r_{ab}^2} \left[(1 - 2\beta + 2\zeta) \frac{1}{r_{bc}} - 2(\beta + \gamma - \zeta) \frac{1}{r_{ac}} \right] - \\ &- (\zeta + \tau) \frac{\Pi_{ab}^{\alpha\lambda}}{r_{ab}^3} (n_{bc\lambda} + n_{ca\lambda}) + (\zeta - \tau) \frac{n_{ab\lambda}}{r_{ab}^2} \frac{\Lambda_{ac}^{\alpha\lambda}}{r_{ac}} + \\ &+ \frac{1}{2} (1 + 2\zeta - 2\tau) \frac{n_{bc\lambda}}{r_{bc}^2} \frac{\Lambda_{ac}^{\alpha\lambda}}{r_{ac}} + 2(1 + \gamma) \frac{n_{bc}^\alpha}{r_{bc}^2 r_{ab}},\end{aligned}\quad (14)$$

where the tensors $\Lambda_{ab}^{\mu\nu} = \gamma^{\mu\nu} + n_{ab}^\mu n_{ab}^\nu$ and $\Pi_{ab}^{\mu\nu} = \gamma^{\mu\nu} + 3n_{ab}^\mu n_{ab}^\nu$ are the projecting and polarizing operators respectively.

The metric tensor (1), the Lagrangian function (4) and the equations of motion (8) define the behavior of the celestial bodies in the post-Newtonian approximation. Hence, they may be used for producing the numerical codes in relativistic orbit determination formalisms for planets and satellites (Moyer, 1981; Ries *et al.*, 1991; Standish *et al.*, 1992) as well as for analysing the gravitational experiments in the solar system (Will, 1993; Pitjeva, 1993; Anderson *et al.*, 1996).

5.2 Metric Tests of Parametrized Gravitation.

By means of a topographic Legendre expansion complete through the second degree and order, the systematic error in Mercury radar ranging has been reduced significantly (Anderson *et al.*, 1995). However, a *Mercury Orbiter* is required before significant improvements in relativity tests become possible. Currently, the precession rate of Mercury's perihelion, in excess of the 530 arcsec per century ("/cy) from planetary perturbations, is 43.13 "/cy with a realistic standard error of 0.14 "/cy (Anderson *et al.*, 1991). After taking into account a small excess precession from the solar oblateness, Anderson *et al.* find that this result is consistent with general relativity. Pitjeva (1993) has obtained a similar result but with a smaller estimated error of 0.052 "/cy. Similarly, attempts to detect a time variation in the gravitational constant G using Mercury's orbital motion have been unsuccessful, again consistent with general relativity. The current result (Pitjeva, 1993) is $\dot{G}/G = (4.7 \pm 4.7) \times 10^{-12} \text{ yr}^{-1}$.

Metric tests utilizing a *Mercury Orbiter* have been studied both at JPL and at the Joint Institute for Laboratory Astrophysics (JILA) and the University of Colorado. The JPL studies, conducted in the 1970's, assumed that orbiter tracking could provide daily measurements (normal points) between the Earth and Mercury centers of mass with a 10 m standard error. A covariance analysis was performed utilizing a 16-parameter model consisting of six orbital elements for Mercury and Earth respectively, the metric relativity parameters β and γ , the solar quadrupole moment $J_{2\odot}$, and the conversion factor A between unit distance (Astronomical Unit) and the distance in meters between Earth and Mercury. It was assumed that no other systematic effects were present, and that the normal-point residuals after removal of the 16-parameter model would be white and Gaussian. The total data interval, assumed equal to two years, corresponded to 730 measurements. Under the assumed random distribution of data, the error on the mean Earth-Mercury distance was $10/\sqrt{730} = 37 \text{ cm}$. The JPL studies showed, based on a covariance analysis, that the primary metric relativity result from a *Mercury Orbiter* mission would be the determination of the parameter γ , the parameter that measures the amount of spatial curvature caused by solar gravitation. The standard error was 0.0006, about a factor of two improvement over the Viking Lander determination. This accuracy reflected the effect of spatial curvature on the propagation of the ranging signal and also its effect on Mercury's orbit, in particular the precession of the perihelion. The error in the metric parameter β and the error in the solar $J_{2\odot}$ were competitive with current results, but not significantly better.

Within the last five years, a more detailed covariance analysis by the JILA group (Ashby *et al.*, 1995) assumed 6 cm ranging accuracy over a data interval of two years, but with only 40 independent measurements of range. Unmodeled systematic errors were accounted for with a modified worst-case error analysis. Even so, the JILA group concluded that a two-order of magnitude improvement was possible in the perihelion advance, the relativistic time delay, and a possible time variation in the gravitational constant G as measured in atomic units. However, the particular orbit proposed by ESA for its 2000 Plus mission was not analyzed. It is almost certain that the potential of the ESA mission lies somewhere between the rather pessimistic JPL error analysis and the JILA analysis of an orbiter mission more nearly optimized for relativity testing.

5.2.1 Mercury's Perihelion Advance.

The secular trend of Mercury's perihelion in the four-dimensional (4D) PPN formalism depends on the linear combination of the PPN parameters γ and β and the solar quadrupole coefficient $J_{2\odot}$ (Nobili and Will, 1986; Heimberger *et al.*, 1990; Will, 1993):

$$\dot{\pi}_{4D} = (2 + 2\gamma - \beta) \frac{GM_{\odot}n_M}{c^2 a_M(1 - e_M^2)} + \frac{3}{4} \left(\frac{R_{\odot}}{a_M} \right)^2 \frac{J_{2\odot}n_M}{(1 - e_M^2)^2} (3 \cos^2 i_M - 1), \quad \text{"/cy} \quad (15a)$$

where a_M, n_M, i_M and e_M are the mean distance, mean motion, inclination and eccentricity of Mercury's orbit. The parameters M_\odot and R_\odot are the solar mass and radius respectively. For the Mercury's orbital parameters one obtains:

$$\dot{\pi}_{4D} = 42''.98 \left[\frac{1}{3}(2 + 2\gamma - \beta) + 0.296 \cdot J_{2\odot} \times 10^4 \right], \quad ''/\text{cy} \quad (15b)$$

Thus, the accuracy of the relativity tests on the *Mercury Orbiter* mission will depend on our knowledge of the solar gravity field. The major source of uncertainty in these measurements is the solar quadrupole moment $J_{2\odot}$. As evidenced by the oblateness of the photosphere (Brown *et al.*, 1989) and perturbations in frequencies of solar oscillations, the internal structure of the Sun is slightly aspherical. The amount of this asphericity is uncertain. It has been suggested that it could be significantly larger than calculated for a simply rotating star, and that the internal rotation rate varies with the solar cycle (Goode and Dziembowski 1991). Solar oscillation data suggest that most of the Sun rotates slightly slower than the surface except possibly for a more rapidly rotating core (Duvall and Harvey, 1984). An independent measurement of $J_{2\odot}$ performed with the *Mercury Orbiter* would provide a valuable direct confirmation of the indirect helioseismology value. Furthermore, there are suggestions of a rapidly rotating core, but helioseismology determinations have been limited by uncertainties for depths below 0.4 solar radii (Libbrecht and Woodard, 1991).

The *Mercury Orbiter* will help us understand this asphericity and independently will enable us to gain some important data on the properties of the solar interior and the features of its rotational motion. Preliminary analysis of a *Mercury Orbiter* mission suggests that $J_{2\odot}$ can be determined to at best $\sim 10^{-9}$ (Ashby, Bender and Wahr, 1995). Even a determination ten times worse, on the order of 10% of the total effect, would be comparable in accuracy to the solar oscillation determination (Brown *et al.*, 1989).

In addition to this, the studies of Mercury's perihelion advance might provide us with an opportunity to test the hypothesis of the multi-dimensionality ($D > 4$) of our physical world (Gross and Perry, 1983; Lim and Wesson, 1992; Kalligas *et al.*, 1995). As it was shown by Lim and Wesson, multi-dimensional extensions of general relativity may produce observable contributions to the four-dimensional physical picture. It has been conjectured that even a small extra (curved) dimension may affect the dominant terms in the four-dimensional metric, thereby altering the theoretical value of the perihelion shift. For example, in the five-dimensional Kaluza-Klein theory, the predicted value of Mercury's perihelion advance is given as

$$\dot{\pi}_{5D} = \dot{\pi}_{4D}(1 + k^2), \quad (15c)$$

where k is the small parameter representing the motion of the body in the extra fifth dimension.

It should be noted that the *Mercury Orbiter* itself, being placed on orbit around Mercury, will experience the phenomenon of periapse advance as well. However, we expect that uncertainties in Mercury's gravity field will mask the relativistic precession, at least at a level of interest for ruling out alternative gravitational theories.

5.2.2 The Precession Phenomena.

In addition to the perihelion advance metric theories predict several precession phenomena associated with the angular momentum of the bodies. Thus, according to PPN formalism, the spin of a gyroscope \vec{s}_0 attached to a test body orbiting around the Sun precesses with respect to a distant standard of rest - quasars or distant galaxies. The theory of Fermi-Walker transport describes the motion of the spin vector of a gyroscope by the relation

$$\frac{d\vec{s}_0}{dt} = [\vec{\Omega} \times \vec{s}_0]. \quad (16)$$

where the angular velocity $\vec{\Omega}$ has the following form (Will, 1993):

$$\vec{\Omega} = -\frac{1}{2}[\vec{v} \times \vec{a}] + (\gamma + \frac{1}{2})[\vec{v} \times \vec{\nabla}U_{\odot}] - (1 + \gamma)\frac{\mu_{\odot}}{2r^3}(\vec{S}_{\odot} - 3(\vec{S}_{\odot}\vec{n})\vec{n}). \quad (17)$$

Here \vec{v} is the velocity of the body, $\vec{a} = d\vec{v}/d\tau - \vec{\nabla}U_{\odot}$ is the non-gravitational acceleration of the body under consideration (for example from the solar radiation pressure), \vec{r} the body's heliocentric radius vector, and U_{\odot} is the gravitational potential of the Sun at the body's location.

The first term in (17) is the Thomas precession. It is a special relativistic effect due to the non-comutativity of nonaligned Lorentz transformations (Thomas, 1927; Bini *et al.*, 1994). It also can be interpreted as a coupling between the bodies velocity and the non-gravitational forces acting on it. The studies of this precession may provide an additional knowledge of the non-gravitational environment of Mercury and its interaction with the solar wind. As for the quasi-Lorentz transformations to the proper coordinate reference frame, this term produces a negligible small contribution to the total precession rate of the satellite's orbit.

The second term is known as a geodetic precession (De-Sitter, 1916). This term arises in any non-homogeneous gravitational field because of the parallel transport of a direction defined by \vec{s}_0 . It can be viewed as spin precession caused by a coupling between the particle velocity \vec{v} and the static part of the space-time geometry (1). For Mercury orbiting the Sun this precession has the form:

$$\vec{\Omega}_G = (\gamma + \frac{1}{2})\frac{\mu_{\odot}}{r^3}(\vec{r} \times \vec{v}), \quad (18)$$

where μ_{\odot} is the solar gravitational constant, r is the distance from Mercury to the Sun and v is its the orbital velocity. This effect should be studied for the *Mercury Orbiter*, which, being placed in orbit around Mercury is in effect a gyroscope orbiting the Sun. Thus, if we introduce the angular momentum per unit mass, $\vec{L} = \vec{r} \times \vec{v}$, of Mercury in solar orbit, the equation (18) shows that $\vec{\Omega}_G$ is directed along the pole of ecliptic, in the direction of \vec{L} . The vector $\vec{\Omega}_G$ has constant part

$$\vec{\Omega}_0 = \frac{1}{2}(1 + 2\gamma)\frac{\mu_{\odot}\omega_M}{a_M} = \frac{1 + 2\gamma}{3} \cdot 0.205 \text{ ''/yr}, \quad (19a)$$

with the significant correction due to the eccentricity e_M of the Mercury's orbit,

$$\vec{\Omega}_1 \cos \omega_M t = \frac{3}{2}(1 + 2\gamma)\frac{\mu_{\odot}\omega_M}{a_M}e_M \cos \omega_M t = \frac{1 + 2\gamma}{3} \cdot 0.126 \cos \omega_M t \text{ ''/yr}, \quad (19b)$$

where ω_M is Mercury's sidereal frequency and t is reckoned from a perihelion passage; a_M is the semi-major axis of Mercury's orbit.

This effect has been studied for the motion of lunar perigee and the existence of the geodetic precession was confirmed with an accuracy of 10% (Bertotti, Ciufolini and Bender, 1987). Two other groups has analyzed the lunar laser-ranging data to estimate the deviation of the lunar orbit from the predictions of general relativity (Shapiro *et al.*, 1988; Dickey *et al.*, 1989). These predictions have been confirmed within the standard deviation of 2%.

Certainly this experiment, being performed in the vicinity of the Mercury, requires complete analysis, taking into account the real orbital parameters of the spacecraft. Our preliminary analysis shows that the precession of the satellites's orbital plane should include a contribution of order 0.205 ''/yr from the geodetic precession. Therefore special studies of this precession should be included in future studies of the *Mercury Orbiter* mission.

The third term in the expression (17) is known as Lense-Thirring precession $\vec{\Omega}_{LT}$. This term gives the relativistic precession of the gyroscope's spin \vec{s}_0 caused by the intrinsic angular momentum \vec{S} of the central body. This effect responsible for a small perturbation of orbits of artificial satellites around the Earth (Tapley *et al.*, 1972; Ries *et al.*, 1991). Unfortunately, preliminary studies have shown that this effect is also negligibly small for the satellite's orbit around Mercury and, moreover, it is masked by the uncertainties of the orbit's inclination.

5.2.3 The Test of the Time Dependence of the Gravitational Constant.

As pointed out by Dirac, the age of the universe, H_0^{-1} , in terms of the atomic unit of time, e^2/mc^3 , is of order 10^{39} , as is the ratio, $e^2/(Gm_pm)$, of the electrical force between the electron and proton to the gravitational force between the same particles. This, according to Dirac, suggested that both ratios are functions of the age of the universe, and that the gravitational constant, G , might vary with time. In order to account for this possibility, and assuming that $\dot{G} = dG/dt$ is constant in first approximation, we define a parameter λ_G as follows: $\dot{G}/G = -\lambda_G H_0$. Then, the effect of λ_G on the motion of bodies in the solar system can be introduced as an additional perturbation in the barycentric equations of motion (8) in the following form:

$$\delta_{\lambda_G} \ddot{\vec{r}} = \lambda_G H_0 \sum_{b \neq a} \frac{m_a t}{r_{ab}^3} \vec{n}_{ab} + \mathcal{O}(t^2). \quad (20)$$

The major contribution to this effect for the motion of celestial bodies in the solar system is the Sun. The importance of the *Mercury Orbiter* mission will be in providing the tracking data necessary for establishing a more accurate determination of the orbital elements of this planet. Then, combined together with other solar system data (Chandler *et al.*, 1994; Anderson *et al.*, 1996), this information will enable one to perform a more accurate test of the hypothesis (20).

5.2.4 The Planetary Test of the Equivalence Principle.

The development of the parameterized post-Newtonian (PPN) formalism, has provided a useful framework for testing the violation of the Strong Equivalence Principle (SEP) for extended bodies (Anderson *et al.*, 1996). In that formalism, the ratio of passive gravitational mass $m_{b(g)}$ to inertial mass $m_{b(i)}$ is given by

$$\frac{m_{b(g)}}{m_{b(i)}} = 1 + \eta \frac{\Omega_b}{m_b c^2}, \quad (21)$$

where m_b is the rest mass of a body b and $\Omega_b = \gamma_{\mu\nu} \Omega_b^{\mu\nu}$ is its gravitational binding energy (2.7). Numerical evaluation of the integral of expression (7) for the standard solar model (Ulrich, 1982) gives

$$\left(\frac{\Omega}{mc^2} \right)_S \approx -3.52 \cdot 10^{-6} \quad (22)$$

The SEP violation is quantified in eq.(21) by the parameter η . In fully-conservative, Lorentz-invariant theories of gravity the SEP parameter is related to the PPN parameters by

$$\eta = 4\beta - \gamma - 3 \quad (23a)$$

and is more generally related to the complete set of PPN parameters through the relation (Will, 1993)

$$\eta = 4\beta - \gamma - 3 - \frac{10}{3} \xi - \alpha_1 + \frac{1}{3} (2\alpha_2 - 2\zeta_1 - \zeta_2). \quad (23b)$$

A difference between gravitational and inertial masses produces observable perturbations in the motion of celestial bodies in the solar system. By analyzing the effect of a non-zero η on the dynamics

of the Earth-Moon system moving in the gravitational field of the Sun, Nordtvedt (1968b) found a polarization of the Moon's orbit in the direction of the Sun with amplitude $\delta r \sim \eta C_0$, where C_0 is a constant of order $13m$ (Nordtvedt effect). The most accurate test of this effect is presently provided by Lunar Laser Ranging (LLR) (Shapiro *et al.*, 1976; Dickey *et al.*, 1989), and in the most recent results (Dickey *et al.*, 1994; Williams *et al.* 1995) the parameter η was determined to be

$$\eta = -0.0005 \pm 0.0011. \quad (24)$$

Also results are available from numerical experiments with combined processing of LLR, spacecraft tracking, planetary radar and Very Long Baseline Interferometer (VLBI) data (Chandler *et al.* 1994). Recently, the analysis of the accuracy for planetary SEP violation in Sun-Jupiter-Mars-Earth system has been done for future Mars missions (Anderson *et al.*, 1996). We would like to emphasize that a measurement of the sun's gravitational to inertial mass ratio can be performed using the Mercury ranging experiment. Indeed, let us consider, for example, the dynamics of the four-body Sun-Mercury-Earth-Jupiter system in the solar system barycentric inertial reference frame. The quasi-Newtonian acceleration of Mercury (M) with respect to the Sun (S), $\ddot{\vec{r}}_{SM}$, is straightforwardly calculated from the equations (8) to be:

$$\ddot{\vec{r}}_{SM} = \ddot{\vec{r}}_M - \ddot{\vec{r}}_S = -m_{SM}^* \cdot \frac{\vec{r}_{SM}}{r_{SM}^3} + m_J \left[\frac{\vec{r}_{JS}}{r_{JS}^3} - \frac{\vec{r}_{JM}}{r_{JE}^3} \right] + \eta \left(\frac{\Omega}{mc^2} \right)_S m_J \frac{\vec{r}_{JS}}{r_{JS}^3}, \quad (25)$$

where $m_{SM}^* \equiv m_S + m_M + \eta \left[m_S \left(\frac{\Omega}{mc^2} \right)_M + m_E \left(\frac{\Omega}{mc^2} \right)_S \right]$. The first and second terms on the right side of the equation (25) are the Newtonian and the tidal acceleration terms respectively. We will denote the last term in this equation as \vec{A}_η . This is the SEP acceleration term which is of order c^{-2} and it is treated as a perturbation on the restricted three-body problem. While it is not the only term of that order, the other post-Newtonian c^{-2} terms (suppressed in eq.(25)) do not affect the determination of η until the second post-Newtonian order ($\sim c^{-4}$). The corresponding SEP effect is evaluated as an alteration of the planetary Keplerian orbit. The subscripts (E) and (J) indicate Earth and Jupiter, respectively. To good approximation the SEP acceleration \vec{A}_η has constant magnitude and points in the direction from Jupiter to the Sun, and because it depends only on the mass distribution in the Sun, the Earth and Mercury experience the same perturbing acceleration. The responses of the trajectories of each of these planets due to the term \vec{A}_η determines the perturbation in the Earth-Mercury range and allows a detection of the SEP parameter η through a ranging experiment.

The presence of the acceleration term \vec{A}_η in the equations of motion (25) results in a polarization of the orbits of Earth and Mercury, exemplifying the planetary SEP effect. We have examined this effect on the orbit of Mercury by carrying out first-order perturbation theory about the zeroth order of Mercury's orbit, taken to be circular. Then the SEP perturbation produces the polarization of the Mercury's orbit of order: $\delta r = \eta C_M$, where C_M is a constant of order 81 m. However, it turns out that the eccentricity correction played a significant role in the similar problem of Mars motion (Anderson *et al.*, 1996). One reason for this is that these corrections include "secular" matrix elements which are proportional to the time t . Such elements dominate at large times, and the eccentricity corrections thereby qualitatively change the nature of the solution in the linear approximation. Thus, for a given value of SEP parameter η , the polarization effects on the Mercury orbit are more than two orders of magnitude larger than on the lunar orbit. The additional result of these studies is that the mass of Jupiter, m_J , can be determined more accurately from a few years of Earth-Mercury ranging than from Pioneer 10,11 and Voyager 1,2 combined. This analysis shows a rich opportunity for obtaining new scientific results from the program of ranging measurements to Mercury during the *Mercury Orbiter* mission. Certainly, for the future analysis of the general planetary SEP violation problem one should modify the theoretical model to include effects due to Venus, Mars and Saturn and perform numerical experiments with combined data collected from the planetary missions, LLR and VLBI.

Moreover, the analysis of *Mercury Orbiter* ranging data might provide the opportunity for another fundamental test, namely a solar system search for dark matter. The data obtained during this mission will allow one to study the conditions leading to the violation of the strong equivalence principle in the motion of extended bodies in the solar system resulting from possible composition, shape and rotation dependent coupling of dark matter to the matter of different astrophysical objects.

5.2.5 The Redshift Experiment.

Another important experiment that could be performed on a *Mercury Orbiter* mission is a test of the solar gravitational redshift, provided a stable frequency standard is flown on the spacecraft. The experiment would provide a fundamental test of the theory of general relativity and the Equivalence Principle upon which it and other metric theories of gravity are based (Will, 1993). Because in general relativity the gravitational redshift of an oscillator or clock depends upon its proximity to a massive body (or more precisely the size of the Newtonian potential at its location), a frequency standard at the location of Mercury would experience a large, measurable redshift due to the Sun. Moreover, the eccentricity of Mercury's orbit would be highly effective in varying the solar potential at the clock, thereby producing a distinguishing signature in the redshift. The anticipated frequency variation between perihelion and aphelion is to first-order in the eccentricity:

$$\left(\frac{\delta f}{f_0}\right)_{e_M} = \frac{2\mu_\odot e_M}{a_M}. \quad (26)$$

This contribution is quite considerable and is calculated to be $(\delta f/f_0)_{e_M} = 1.1 \times 10^{-8}$. Its absolute magnitude, for instance, at the wave length $\lambda_0 = 3$ cm ($f_0 = 10$ GHz) is $(\delta f)_{e_M} = 110$ Hz. We would also benefit from the short orbital period of Mercury, which would permit the redshift signature of the Sun to be measured several times over the duration of the mission. Depending upon the stability of the frequency standard, a *Mercury Orbiter* redshift experiment could provide a substantial improvement over previous tests of the solar redshift.

At this time, there exist two comparable tests of the solar redshift. The first test was performed by using the well-known technique of measuring shifts in the positions of spectral lines of atoms in the solar atmosphere. Recent measurements of an oxygen triplet have verified the solar redshift predicted by general relativity to an accuracy of 1% (LoPresto *et al.*, 1991). The first test of the solar redshift with an interplanetary probe was performed recently with the *Galileo* spacecraft during its mission to Jupiter (Krisher, 1993). In order to obtain gravity-assists from the Earth and Venus, the spacecraft traveled on a trajectory that took it in and out of the solar potential. By measuring the frequency of an ultra-stable crystal oscillator (USO) flown on the *Galileo* spacecraft during this phase of the mission, the redshift prediction was again verified to an accuracy of 1%. The accuracy obtained was limited by the long-term stability of the USO. After calibration of systematic errors (e.g., a linear drift due to aging), the fractional frequency stability was roughly 10^{-12} .

During a *Mercury Orbiter* mission, the variation in the solar potential between perihelion and aphelion would produce a redshift of roughly 1 part in 10^8 according to general relativity (for a detailed analysis of the anticipated redshift, see Krisher, 1993). With a fractional frequency stability of 10^{-12} , we could test the redshift with an accuracy of 1 part in 10^4 (or 0.01%). This level of accuracy is comparable to that obtained by the 1976 Vessot/NASA Gravity-Probe A (GP-A) experiment performed in the gravitational field of the Earth with a hydrogen maser on a Scout rocket (Vessot *et al.*, 1980). Greater sensitivity could be obtained if an atomic frequency standard were flown on *Mercury Orbiter*. An atomic standard having a typical stability of 1 part in 10^{15} would permit the redshift to be tested to 1 part in 10^7 .

5.2.6 The Shapiro Time Delay.

According to general relativity, an electromagnetic signal propagating near the Sun will be delayed by solar gravity (Shapiro, 1964). In PPN formalism this additional coordinate time delay, $\Delta\tau$, is given by (Will, 1993; Lebach *et al.*, 1995)

$$c\Delta\tau = \mu_{\odot} \left[(1 + \gamma) \ln \left(\frac{r_E + r_p + r_{Ep}}{r_E + r_p - r_{Ep}} \right) - \frac{\tau}{2r_{Ep}} \left(\frac{r_E^2 - r_p^2 - r_{Ep}^2}{r_p} + \frac{r_p^2 - r_E^2 - r_{Ep}^2}{r_E} \right) \right] \quad (27)$$

where r_E and r_p are the respective distances of the Earth and the planet from the Sun, and $\vec{r}_{Ep} = \vec{r}_p - \vec{r}_E$ is the earth-planet distance. At superior conjunction this formula can be approximated by

$$c\Delta\tau = \mu_{\odot} \left[(1 + \gamma) \ln \left(\frac{4r_E r_p}{d^2} \right) + 2\tau \right] \quad (28)$$

where d is the closest distance from the center of the Sun to the beam trajectory.

By ranging to *Mariner*, *Viking*, and *Voyager* spacecraft during solar conjunction, it has been possible to test this effect to a few percent or better (Krisher *et al.*, 1991 and references therein). From maximum elongation to superior conjunction, $\Delta\tau$ increases from 15 to 240 microseconds for Mercury. However, all previous spacecraft tests have depended on an S-band uplink. The *Mercury Orbiter* may provide the first opportunity for an X-band test. The higher frequency, by the factor 11/3, will reduce the systematic error from propagation of the radio beam through the solar corona. It will also be possible to perform a Doppler relativity test as suggested by Bertotti and Giampieri (1992). The multi conjunctions may be useful in constraining certain alternative theories of gravitation involving anisotropic metric potentials. The Doppler shift caused by solar gravity could depend on the geometry of the conjunction.

6 Radio Science Instrumentation

The *Mercury Orbiter* radio science objectives require on board instrumentation with the following capabilities:

1. Up-link at X-band (~ 3.5 cm wavelength) from ground stations to the spacecraft.
2. X-band down-link coherently referenced to the up-link signal.
3. Crystal or atomic ultra-stable oscillator (USO).
4. Simultaneous down-link at S (~ 13 cm) and X-bands referenced to the USO.

Any S-band up-link capability, perhaps provided for commanding or emergency communications, would not be used during radio science observations. The S-band up-link provides far superior performance. Up-link and down-link at higher frequencies (i.g. KA-band) should be considered for the *Mercury Orbiter* mission for purposes of enhancing the gravity and relativity measurements. A multi-link Doppler systems should be considered as well. Oven controlled USO's has been included in several radio science systems in the past Including *Voyager*, *Galileo* and *Cassini*. The *Mercury Orbiter* mission is far enough in the future that the crystal oscillator will be replaced by an atomic standard.

Starting with an X-band system, the cost of adding a dual-frequency downlink capability to enhance radio propagation measurements would require about 15-20 watts of additional power and 4-5 kg of additional mass for an S-band transmitter. Observations could then be performed at both occultation ingress and egress by transmitting both frequencies coherently from a common on-board oscillator. It would not be necessary to add a special ultra-stable oscillator (USO) for this purpose as long as the downlinks are phase coherent.

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Appendix A: The astrophysical parameters used in the paper.

Solar radius : $R_{\odot} = 695,980$ km,

Solar gravitational constant : $\mu_{\odot} = \frac{GM_{\odot}}{c^2} = 1.4766$ km,

Solar quadrupole coefficient (Brown *et al.*, 1989) : $J_{2\odot} = (1.7 \pm 0.17) \times 10^{-7}$,

Solar rotation period : $\tau_{\odot} = 25.36$ days,

Mercury's mean distance : $a_M = 0.3870984$ AU = 57.91×10^6 km,

Mercury's radius : $R_M = 2,439$ km,

Mercury's gravitational constant : $\mu_M = \frac{GM_M}{c^2} = 1.695 \times 10^{-7} \mu_{\odot}$,

Mercury's sidereal period : $T_M = 0.241$ yr = 87.96 days,

Mercury's rotational period : $\tau_M = 59.7$ days ,

Eccentricity of Mercury's orbit : $e_M = 0.20561421$,

Jupiter's gravitational constant : $\mu_J = 9.547 \times 10^{-4} \mu_{\odot}$,

Jupiter's sidereal period : $T_J = 11.865$ yr,

Astronomical Unit : $AU = 1.49597892(1) \times 10^{13}$ cm,

9 Tables

Table 1

Mercury Radar-Ranging Measurements		
Timespan	Antenna	Number of Observations
1967-1971	Arecibo	85
1966-1971	Haystack	217
1971-1974	Goldstone	38
1974-1975	Mariner 10	2
1978-1982	Arecibo	157
1986-1990	Goldstone	132